

# SPACE-TIME UNCERTAINTY FROM HIGHER-DIMENSIONAL DETERMINISM

(Or: How Heisenberg was right in 4D because Einstein  
was right in 5D)

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## Abstract

Heisenberg's uncertainty relation is commonly regarded as defining a level of unpredictability that is fundamentally incompatible with the deterministic laws embodied in classical field theories such as Einstein's general relativity. We here show that this is not necessarily the case. Using 5D as an example of dimensionally-extended relativity, we employ a novel metric to derive the standard quantum rule for the action and a form of Heisenberg's relation that applies to real and virtual particles. The philosophical implications of these technical results are somewhat profound.

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Einstein and Heisenberg espoused fundamentally different views of mechanics, the former arguing that God does not play dice with the world, and the latter seeing indeterminacy as an essential aspect of it. This problem has recently come to the fore again as part of new attempts to unify gravity with the interactions of particle physics. The best route to unification is commonly regarded as dimensional extension, wherein four-dimensional (4D) spacetime is augmented by extra parts<sup>1</sup>. Hence 5D induced-matter and membrane theory (both modern versions of Kaluza-Klein theory), 10D

supersymmetry, 11D supergravity and 26D string theory. Does the algebraic richness of these new theories offer a way out of the old physical problem?

Mechanics is often regarded as a staid subject, but it is actually full of conceptual twists that offer clues to how we might answer the above question<sup>2</sup>. Newton's mechanics defines force as the product of mass and acceleration, where the latter is calculated from separate space ( $x$ ) and ( $t$ ) coordinates, while the former is just given. Mach's mechanics postulates that the local (inertial rest) mass  $m$  of a particle ought to be calculable as the sum of other influences in the universe; but while this principle motivated Einstein, it was not properly incorporated into general relativity. The latter is a curved-space version of special relativity, whose underlying framework is due to Minkowski. He realized that the speed of light allows the time to be treated as a coordinate that is on the same footing as those of space ( $x^\alpha = ct, x, y, z$ ). Both versions of relativity are classical field theories, built along the lines of Maxwell's theory of electromagnetism, and neither incorporates at a basic level the quantum of action  $h$  named after Planck. Indeed, an examination of the foundations of classical and quantum mechanics shows that the former is a theory of accelerations while the latter is a theory of changes in momentum or forces. The two concepts overlap, of course, but are only

equivalent when the mass is constant. Examples where this is not so involve the acceleration of a rocket as it burns fuel and leaves the Earth,<sup>2</sup> or the process by which particles gain mass by a Higgs-type scalar field in the early universe.<sup>3</sup> This difference has been commented on by a number of astute workers, and reflects our lack of a theory of the origin of mass.<sup>4</sup> This is pointed up by the fact that there is an ambiguity between the mass of a particle and the energy of a resonance in particle interactions, and that both are related via a Heisenberg-type relation with their lifetimes. In relativity as applied to either classical or quantum situations, the ordinary time  $t$  is replaced by the proper time  $\tau$  of a flat Minkowski metric or the interval  $s$  of a curved Riemannian metric. The latter is defined by  $ds^2 = g_{\alpha\beta}dx^\alpha dx^\beta$ . (Here  $g_{\alpha\beta}$  are the components of the metric tensor, which are basically gravitational potentials, and there is a sum over the repeated indices  $\alpha = 0, 1, 2, 3$ .) There are two equivalent ways to obtain the dynamics of a particle in such a metric-based theory. One involves forming the path length  $(\int ds)$ , and using a version of Fermat's theorem to minimize this ( $\delta [\int ds] = 0$ ), which results in the four components of the geodesic equation. The other way is to form a Lagrangian density from  $s$  or a function of it, employ the Euler-Lagrange equations, and so obtain the four equations of motion. [In both approaches,

the energy of a particle and its momentum in ordinary space are associated with the  $t, xyz$  or  $0, 123$  components.] However, these approaches necessarily result in equations for the acceleration, because the mass  $m$  is absent. By contrast, the action of 4D particle physics ( $\int mcds$ ) involves the mass explicitly. This mismatch in the ways in which dynamics is set up becomes acute when we attempt to extend the manifold in such a way as to account for the mass-insensitive nature of gravity and the mass-sensitive nature of particle interactions.

Fortunately, we are aided by some new and startling results from 5D relativity. There are two popular versions of this. Induced-matter theory views mass as a direct manifestation of an unconstrained fifth dimension, and its corpus of technical results is largely based on the canonical metric.<sup>5,6</sup> Membrane theory views matter as confined to a surface in a 5D world, and most of its technical results are based on the warp metric.<sup>7,8</sup> Both theories regard 4D spacetime as embedded in a 5D manifold whose line element is defined by  $dS^2 = g_{AB}dx^A dx^B$  ( $A, B = 0, 123, 4$  where the extra coordinate will henceforth be labelled  $x^4 = l$ ). Both theories are well-regarded because they represent the basic extension of general relativity and the low-energy limit of higher- $N$  accounts. In addition, it has recently been shown that the

field equations of these theories are in fact equivalent, so their solutions for the potentials  $g_{AB}$  are common.<sup>9</sup> We will not be concerned with the field equations in what follows, but mention two results to do with dynamics that are also common: (a) Massive particles travelling on timelike geodesics in 4D ( $ds^2 > 0$ ) can be regarded as travelling on null geodesics in 5D ( $dS^2 = 0$ ).<sup>10,11</sup> In other words, what we regard as ordinary objects in spacetime are like photons in the bigger manifold, and they are in causal contact. This has obvious implications for the wave-like behaviour of particles, such as those of electrons in the classic double-slit experiment. (b) Massive particles travelling along  $s$ -paths in 4D in general change their mass via  $m = m(s)$ , and this is connected to the existence of a fifth force which is due to the fifth dimension.<sup>12,13</sup> That such a new force exists can be readily seen, by noting that in theories like general relativity there is an orthogonality condition which relates the 4-velocities ( $u^\alpha \equiv dx^\alpha/ds$ ) to the components of the acceleration or force per unit mass ( $f_\alpha$ ), namely  $u^\alpha f_\alpha = 0$ . But in 5D, the corresponding relation is  $u^A f_A = 0$ , so perforce  $u^\alpha f_\alpha = -u^4 f_4 \neq 0$ . Also, the new force acts parallel to the 4-velocity. Provided we use the proper time  $s$  to parametrize the motion, and seek to make contact with the large amount of data we already have on 4D dynamics, the logical way to quantify

this (presumably small) force is via a variation in the mass. [Depending on whether one uses the canonical or warp metric, the mass itself is either the coordinate or its rate of change, but these two identifications are essentially equivalent for null 5D geodesics, as will be noted below.] A force which acts parallel to the 4-velocity, and changes the mass and therefore the momentum, is new to classical field theory but not to quantum theory. Is there a link? We conjecture that  $u^\alpha f_\alpha \neq 0$  is related to  $dx^\alpha dp_\alpha \neq 0$ ; and that the classical laws of an extended manifold are related to the quantum uncertainty of spacetime.

We now proceed to show, in short order, how to derive Heisenberg's uncertainty relation in 4D from Einstein-like deterministic dynamics in 5D. We will use the Lagrangian approach, but the results are compatible with a longer approach based on the geodesic equation<sup>14,15</sup>. Conventional units are retained for ease of physical interpretation. We will use the notation of induced-matter theory as opposed to membrane theory, because it is more direct. However, we introduce a new form for the line element, which for reasons that will become apparent we refer to as the Planck gauge:

$$dS^2 = \frac{L^2}{l^2} g_{\alpha\beta}(x^\gamma, l) dx^\alpha dx^\beta - \frac{L^4}{l^4} dl^2 \quad . \quad (1)$$

Here, we have used 4 of the 5 available degrees of coordinate freedom to

remove the  $g_{4\alpha}$  components of the metric that are traditionally associated with electromagnetism; and we have used the fifth degree of coordinate freedom to set  $g_{44} = -L^4/l^4$ . Algebraically, (1) is therefore general, insofar as its 4D part has been factorized by  $L^2/l^2$  but  $g_{\alpha\beta} = g_{\alpha\beta}(x^\gamma, l)$  is still free. Physically, however, we do not expect any incursion of the fifth dimension into 4D spacetime, since this would violate the Weak Equivalence Principle<sup>1,2</sup>. We therefore proceed with  $g_{\alpha\beta} = g_{\alpha\beta}(x^\gamma)$ . Then, it may be mentioned in passing that the coordinate transformation (change of gauge)  $l \rightarrow L^2/l$  converts (1) to the canonical metric of induced-matter theory<sup>5,6</sup>, and that a further transformation converts it to the warp metric of membrane theory.<sup>7,8</sup> The constant  $L$  in (1) is required by dimensional consistency, and corresponds physically to the characteristic size of the potential in the fifth dimension. However, it also sets a scale for 4D spacetime. [For the whole universe, a reduction of the field equations shows that  $L = (3/\Lambda)^{1/2}$  where  $\Lambda$  is the cosmological constant.<sup>1]</sup> From (1), we can form the Lagrangian density  $(dS/ds)^2$  and use the Euler-Lagrange equations to obtain the associated 5-momenta in the standard manner. These 5-momenta define a 5D scalar which is the analog of the one used in 4D quantum mechanics. Following the



philosophy outlined above, we split it into its 4D and fifth parts as follows:

$$\int P_A dx^A = \int (P_\alpha dx^\alpha + P_l dl) = \int \frac{2L^2}{l^2} \left[ 1 - \left( \frac{L}{l} \frac{dl}{ds} \right)^2 \right] ds \quad . \quad (2)$$

However, this is actually zero for particles moving on null geodesics of (1) with  $dS^2 = 0$ , as for induced-matter and membrane theory. Then  $l = l_0 \exp(\pm s/L)$  and  $dl/ds = \pm (l/L)$ , so the variation is slow for  $s/L \ll 1$  and the mass parametrizations for the two theories are equivalent modulo  $L$ , as noted above. For (1), the appropriate parametrization is clearly  $l = h/mc$ , which means that the extra coordinate is the Compton wavelength of the particle. The first part of (1) then gives the conventional action of 4D particle physics. But note the important difference that while the 4D action is finite and describes a particle with finite energy, the 5D action is zero because  $\int P_A dx^A = 0$ .

The quantity that corresponds to this in 4D is

$$\int p_\alpha dx^\alpha = \int m u_\alpha dx^\alpha = \int \frac{h ds}{cl} = \pm \frac{h}{c} \frac{L}{l} \quad . \quad (3)$$

The sign choice here goes back to the reversible nature of the motion in the fifth dimension, but this is unimportant so we suppress it in what follows. We also put  $L/l = n$ , to make contact with other work on the wave nature

of particles. Then (3) says

$$\int mcds = nh \quad , \quad (4)$$

which of course is known to every physics student. More revealingly, from previous relations we can form the scalar

$$dp_\alpha dx^\alpha = \frac{h}{c} \left( \frac{du_\alpha}{ds} \frac{dx^\alpha}{ds} - \frac{1}{l} \frac{dl}{ds} \right) \frac{ds^2}{l} \quad . \quad (5)$$

Here the first term is conventional, and is zero if the acceleration is zero and / or if the standard orthogonality relation  $f_\alpha u^\alpha = 0$  holds (see above). But the second term is unconventional, and is in general non-zero. It measures the variation in the (inertial rest) mass of a particle required to balance the extended laws of conservation. The anomalous contribution has magnitude

$$|dp_\alpha dx^\alpha| = \frac{h}{c} \left| \frac{dl}{ds} \right| \frac{ds^2}{l^2} = \frac{h}{c} \frac{ds^2}{Ll} = n \frac{h}{c} \left( \frac{dl}{l} \right)^2 \quad . \quad (6)$$

This is a Heisenberg-type relation. Reading it from right to left, it says that the fractional change in the position of the particle in the fifth dimension has to be matched by a change of its dynamical quantities in 4D spacetime. We can write (6) in more familiar form as

$$|dp_\alpha dx^\alpha| = \frac{h}{c} \frac{dn^2}{n} \quad . \quad (7)$$

This is as far as the formal analysis goes. We have not so far considered questions of topology, but there are clearly two cases of (7) which depend on this. For a particle trapped in a potential box, the Compton wavelength cannot exceed the confining size of the geometry, so  $l \leq L$ ,  $n \geq 1$  and we have a violation of the Heisenberg rule as for virtual particles. For a particle that is free, the Compton wavelength is unconstrained, so  $l > L$ ,  $n < 1$  and we have the conventional Heisenberg rule as for real particles.

In conclusion, we review our technical results and make some philosophical comments. In establishing a consistent scheme for dynamics in more than 4 dimensions, we have been guided by the history of mechanics and have chosen to make contact with extant results by using the proper 4D time as parameter. Of the infinite number of metrics or gauges available, we have proposed that (1) is the most appropriate for applications to particle physics. Inspection shows that this works well because it incorporates the (inertial rest) mass as a coordinate in such a way as to convert the metric to a momentum manifold. This allows the theory of accelerations as used in classical field theories like general relativity to be extended to a theory of momentum changes (forces) as used in quantum theory. The main results are the standard quantization rule for the action (4) and a Heisenberg-type

uncertainty relation (7). An insight from the latter is that virtual and real particles are aspects of the same underlying dynamics, separated by a number  $0 < n < \infty$  at  $n = 1$ . These technical results are neat (and clearly beg for more investigation); but the philosophical implications of the preceding outline are more profound.

Since the 1930s, the view has become ingrained that quantum physics necessarily involves a level of uncertainty or non-predictability that is in conflict with the deterministic laws of classical field theory as represented by Maxwell's electromagnetism and Einstein's general relativity. Surprisingly, we now see that this is not necessarily the case. The dichotomy may be an artificial one, laid on us by our ignorance of the extent of the real world. In a classical field theory like general relativity which is extended to  $N(> 4)$  dimensions, the extra parts of deterministic laws can manifest themselves in spacetime as "anomalous" effects. This will perforce happen in any  $ND$  field theory that is not artificially constrained by "symmetries". The last word has, unfortunately, been used all too often as an excuse for algebraic shortcuts that are not respected by reality. A prime example is the "cylinder condition" of old 5D Kaluza-Klein theory. By removing dependency on the extra coordinate, it emasculates the metric and runs the theory into

insurmountable difficulties to do with the masses of elementary particles and the energy density of the vacuum (the hierarchy and cosmological-constant problems). The modern versions of 5D relativity, namely induced-matter theory and membrane theory, drop the cylinder condition. These theories may be daunting algebraically, but they gain in being richer physically. The same lesson can be applied to 10D supersymmetry, 11D supergravity and 26D string theory. We should recall, in this regard, that Einstein in his later work endorsed higher dimensions and remained adamant that quantum uncertainty was philosophically unacceptable. The results we have shown here could provisionally be used to paraphrase Einstein: “God does not play dice in a higher-dimensional world.”

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